

# Poster: Hybrid Multi-Contact Dynamics for Wedge Jumping Locomotion Behaviors

Ye Zhao, Donghyun Kim, Gray Thomas, and Luis Sentis  
Human Centered Robotics Laboratory, The University of Texas at Austin  
Austin, Texas, USA

yezha@utexas.edu, dk6587@utexas.edu, gray.c.thomas@gmail.com,  
lsentis@austin.utexas.edu.

## ABSTRACT

Legged robots naturally exhibit continuous and discrete dynamics when maneuvering over level-ground and uneven terrains. In recent years, numerous studies have focused on locomotion hybrid dynamics. However, locomotion on more challenging terrains such as split wedges in Figure 1 has rarely been explored, let alone its hybrid dynamics. In this study, we specifically focus on a two-phase hybrid automaton formulation for this highly steep wedge locomotion. This automaton incorporates both multi-contact and flight single contact phase motions. To dynamically balance and jump upwards on this wedge, an aperiodic phase space planning is used for trajectory generations. Three control strategies are employed simultaneously: internal force control, linear and angular momentum control. Finally, simulation results are shown to verify our strategy's effectiveness.

## 1. INTRODUCTION

Although rough terrain locomotion has gained extensive attentions in recent years, walking on extreme terrains like a split wedge-like structure in Figure 1 is still an open area to be explored. This motivates us to study the wedge motion, which spontaneously involves multi-contact dynamics and hybrid dynamics [1]. We formulate a wedge jumping automaton by incorporating both of these dynamics. Our main motivations lie in planning and control strategies. Different from periodic orbit based locomotion such as Poincare return map [1], this study uses a non-periodic phase space planning [4], suitable for highly irregular terrains. As to control strategy, we use whole body operational space control [2] to achieve compliant locomotion motions. Specifically, we realize that properly using angular momentum and internal forces greatly contributes to dynamically stable motion on this almost-vertical terrain. Without special concerns of these two factors, the biped fails to take a step or even balance on this wedge. In this paper, we briefly report our preliminary results and current work in progress.

## 2. HYBRID AUTOMATA

Permission to make digital or hard copies of part or all of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. Copyrights for third-party components of this work must be honored. For all other uses, contact the Owner/Author.

Copyright is held by the owner/author(s).

HSCC '15, Apr 14-16, 2015, Seattle, WA, USA

Copyright 2015 ACM ACM 978-1-4503-3433-4/15/04

<http://dx.doi.org/10.1145/2728606.2728645>

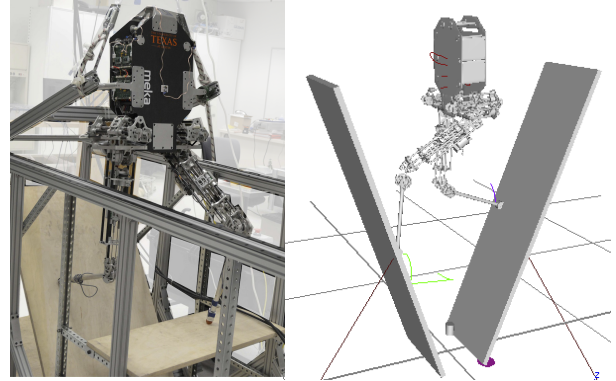


Figure 1: Hume Bipedal Robot Balances on A Steep Wedge Terrain.

A hybrid system is composed of both continuous and discrete dynamics. The continuous dynamics are described by non-linear ordinary differential equations  $\dot{X}(t) = \mathcal{F}(X(t))$ ,  $X(0) \in \mathcal{I}$ , where  $X \in \mathcal{X}$  are system states.  $\mathcal{F} : \mathcal{D} \rightarrow \mathbb{R}^n$  is a vector field on domain  $\mathcal{D}$ . The discrete dynamics are expressed as  $(\mathcal{Q}, \mathcal{I}, \Delta)$ , where  $\mathcal{Q} = \{q_1, q_2, \dots, q_n\}$  represents a set of discrete states;  $\mathcal{I} \subseteq \mathcal{Q} \times \mathcal{X}$  represents an initial condition and  $\Delta : \mathcal{Q} \rightarrow 2^{\mathcal{Q}}$  is discrete transition function. Now let us define a hybrid automaton as follows.

**Definition** A hybrid automaton  $\mathcal{H}$  is a dynamical system, described by a collection [3]

$$\mathcal{H} = (\mathcal{Q}, \mathcal{X}, \mathcal{F}, \mathcal{I}, \mathcal{D}, \mathcal{E}, \mathcal{G}, \mathcal{R}) \quad (1)$$

where  $\mathcal{D}(\cdot) : \mathcal{Q} \rightarrow 2^{\mathcal{X}}$  is a domain;  $\mathcal{F}$  is assumed to be sufficiently smooth in domain  $\mathcal{D}$  and globally Lipschitz continuous in state  $X$ ;  $\mathcal{E} \subseteq \mathcal{Q} \times \mathcal{Q}$  represents a set of edges;  $\mathcal{G}(\cdot) : \mathcal{E} \rightarrow 2^{\mathcal{X}}$  is a guard condition;  $\mathcal{R}(\cdot, \cdot) : \mathcal{E} \times \mathcal{X} \rightarrow 2^{\mathcal{X}}$  is a reset map. This reset map is often referred as an impulse effect in locomotion field. Given an initial condition  $(q_0, X_0) \in \mathcal{I}$ , continuous state  $X$  flows within a domain  $\mathcal{D}(q_0)$ .  $X$  evolves till hitting a guard  $\mathcal{G}(q_0, q_1) \subseteq \mathbb{R}^n$  of an edge  $(q_0, q_1) \in \mathcal{E}$ . Then the discrete state  $q$  switches while  $X$  instantaneously is reset by a map  $\mathcal{R}(q_0, q_1, X) \subseteq \mathbb{R}^n$ .

## 3. HYBRID WEDGE JUMPING DYNAMICS

Based on the general hybrid automaton  $\mathcal{H}$  defined above, we formulate a specific hybrid automaton in Figure 2 for wedge jumping behavior.

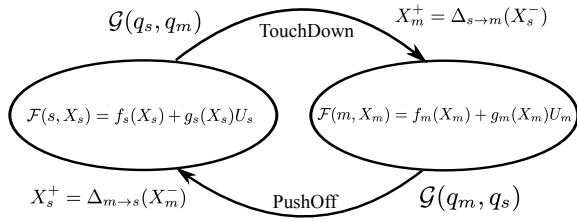


Figure 2: Hybrid Wedge Jumping Automata.

- Discrete state  $\mathcal{Q} = \{q_s, q_m\}$ : flight single contact  $q_s$  and multi-contact  $q_m$ ;
- Continuous state  $\mathcal{X}$ . The Cartesian space,  $\mathcal{C}$  is a fourth-dimensional manifold,  $c := (x, z, \phi, \theta)^T$ , i.e., CoM sagittal, vertical positions, torso roll and pitch angles. Then the state vector is defined as  $X := (c; \dot{c}) \in \mathcal{TC}$ , where  $\mathcal{TC}$  is the tangent bundle of  $\mathcal{C}$ .
- Vector field  $\mathcal{F}(s, X_s) = f_s(X_s) + g_s(X_s)U_s$  (single contact) and  $\mathcal{F}(m, X_m) = f_m(X_m) + g_m(X_m)U_m$  (multi-contact);
- Initial condition  $\mathcal{I} = \{m\} \times \{(X, U) \in \mathcal{TC}_m \times \mathcal{U}_m | mc\}$ , where  $mc$  means "multi-contact"; It is obvious to assume initial condition of wedge motion is multi-contact.
- Domain  $\mathcal{D}(q_s) = \{(X, U) \in \mathcal{TC}_s \times \mathcal{U}_m | sc\}$  ( $sc$  is single contact),  $\mathcal{D}(q_m) = \{(X, U) \in \mathcal{TC}_m \times \mathcal{U}_m | mc\}$ ;
- Edge  $\mathcal{E} = \{(q_s, q_m), (q_m, q_s)\}$ ;
- Guard  $\mathcal{G}(q_s, q_m) = \{(X, U) \in \mathcal{TC}_s \times \mathcal{U}_s | sl\}$  ( $sl$  is "swing leg landing"),  $\mathcal{G}(q_m, q_s) = \{(X, U) \in \mathcal{TC}_m \times \mathcal{U}_m | ss\}$  ( $ss$  is "stance leg starting to swing");
- Reset map  $\mathcal{R}(q_s, q_m, X_m) = \{\Delta_{s \rightarrow m}(X^-)\}$  (from single to multi-contact),  $\mathcal{R}(q_m, q_s, X_s) = \{\Delta_{m \rightarrow s}(X_m^-)\}$  (from multi- to single contact).

The complete hybrid dynamics for wedge jumping comprise two sets of continuous dynamics with reset maps.

$$\Sigma_s : \begin{cases} \dot{X}_s = f_s(X_s) + g_s(X_s)U_s & X_s^- \notin \mathcal{G}(q_s, q_m) \\ X_m^+ = \Delta_{s \rightarrow m}(X_s^-) & X_s^- \in \mathcal{G}(q_s, q_m) \end{cases}$$

$$\Sigma_m : \begin{cases} \dot{X}_m = f_m(X_m) + g_m(X_m)U_m & X_m^- \notin \mathcal{G}(q_m, q_s) \\ X_s^+ = \Delta_{m \rightarrow s}(X_m^-) & X_m^- \in \mathcal{G}(q_m, q_s) \end{cases}$$

#### 4. PRELIMINARY RESULT

According to hybrid dynamics above, we simulate wedge jumping motions, which are shown in Figure 3. The wedge has a  $70^\circ$  slope, which is the currently achieved maximum slope. Prismatic inverted pendulum dynamics are used for our phase space planning [4]. Specifically, hybrid contact transitions between jumping and leading maneuvers are decided by intersections of adjacent phase curves. To successfully jumping over this terrain, the planner incorporates human-mimic motions, such as body moving backwards against the wall to gain momentum energy and bending forwards to compensate linear momentum. This wedge terrain is so vertical that it requires highly non-periodic planner, which is

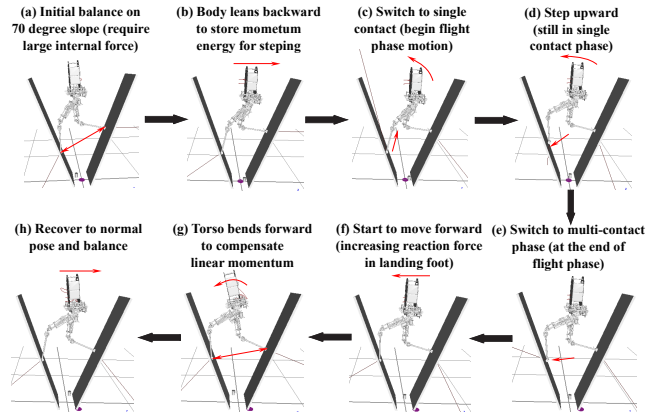


Figure 3: Hybrid Wedge Jumping Behavior with Multi-contact Dynamics. Hybrid dynamics occur during transitions between flight phase and multi-contact phase.

achievable by our generic phase space planning. Numerical integration is adapted to solve this non closed-form state space trajectories. Multi-contact dynamics are leveraged to search reachable space and modulate internal tension behaviors. Then planned CoM and foot trajectories are set as Cartesian tasks of our Hume bipedal robot controlled by whole body operational space control. To achieve a stable stepping after contact occurs, torso pitch orientation is modulated and an approximately 200 N internal force is exerted between two contact feet to satisfy friction cone constraint. In particular, we realize that properly controlling internal force during multi-contact phase (in Figure 3 (a) and (g)) and angular momentum (in Figure 3 (g)) is pivot to achieve this dynamically stable jumping motion.

#### 5. ON-GOING WORK

Our current work includes: (I) Proposing a phase space reachability analysis by incorporating robot physical and environmental constraints. We target increasing the flexibility of our planning strategy to more generally constrained and unconstructed environments, such as a conic terrain; (II) Exploring how prioritized control of internal forces, linear and angular momentum affects extreme locomotion capabilities and performance; (III) Evaluating physical limits of extreme locomotion such as jumping and climbing efficiency, maximum jumping capabilities.

#### 6. REFERENCES

- [1] J. Grizzle, C. Chevallereau, A. D. Ames, and R. W. Sinnet. 3d bipedal robotic walking: Models, feedback control, and open problems. In *Proceedings of IFAC Symposium on Nonlinear Control Systems*, 2010.
- [2] D. Kim, Y. Zhao, G. Thomas, and L. Sentis. Assessing whole-body operational space control in a point-foot series elastic biped: Balance on split terrain and undirected walking. *Submitted*, 2015.
- [3] J. Lygeros, C. Tomlin, and S. Sastry. Hybrid systems: modeling, analysis and control. *preprint*, 1999.
- [4] Y. Zhao and L. Sentis. A three dimensional foot placement planner for locomotion in very rough terrains. In *Humanoid Robots (Humanoids), IEEE-RAS International Conference on*, 2012.